280. Proposed by WILLIAM HOOVER, Ph. D., Athens, Ohio.

On any diameter of a given ellipse is taken a point such that the tangents from it intercept on the tangent at one end of the diameter a length equal to the diameter; the ellipse being $a^2y^2+b^2x^2-a^2b^2=0$. Prove that the locus of the point is $\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^2=\left(\frac{a^2+b^2}{a^2-b^2}\right)^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)$.

Solution by the PROPOSER.

Let the diameter through any point $P(x_1, y_1)$ of the locus cut the ellipse in A, A'. The chord RS of contact of P and the tangent at A' are parallel to the diameter at A_1A_2 conjugate to AA': and then

$$\frac{PB}{RS}=\frac{PA'}{TQ}\text{.........}(1),$$

B being the intersection of RS and PA'. The equation to RS is

$$y=-\frac{b^2x_1}{a^2y_1}x+\frac{b^2}{y_1}\text{.........}(2).$$

If $y=mx+c$.........(3), is the equation to any chord of $a^2y^2+b^2x^2=a^2b^2$.........(4), the length of chord is $l=2ab\surd[(1+m^2)(m^2a^2+b^2-c^2)]\div(m^2a^2+b^2)$.........(5). Comparing (2) and (3), $m=-\frac{b^2x_1}{a^2y_1}$.........(6), $c=\frac{b^2}{y_1}$.........(7). Substituting (6) and (7) in (5), we have

$$RS=2\surd[(a^4y_1^2+b^4x_1^2)(b^2x_1^2+a^2y_1^2-a^2b^2)]\div(a^2y_1^2+b^2x_1^2)\text{.........}(8).$$

OP or $y=\frac{y_1}{x_1}x$.........(9) cuts the curve in

$$x'=-\frac{abx_1}{\surd(a^2y_1^2+b^2x_1^2)},\ y'=-\frac{aby_1}{\surd(a^2y_1^2+b^2x_1^2)}\text{.........}(10).$$

(9) cuts (2) in B, or

$$x''=\frac{abx_1}{\surd(a^2y_1^2+b^2x_1^2)},\ y''=\frac{aby_1}{\surd(a^2y_1^2+b^2x_1^3)}\text{.........}(11).$$

Then $\overline{PB}^2=(x_1^2+y_1^2)(a^2y_1^2+b^2x_1^2-a^2b^2)^2\div(a^2y_1^2+b^2x_1^2)^2$.........(12),
$\overline{PA'}^2=(x_1^2+y_1^2)(a^2y_1^2+b^2x_1^2+a^2b^2)^2\div(a^2y_1^2+b^2x_1^2)$.........(13),
$\overline{TQ}^2=4a^2b^2(x_1^2+y_1^2)\div(a^2y_1^2+b^2x_1^2)$.........(14).

Squaring (1), substituting (8), (12), (13), and reducing, we have

$$a^2b^2(x_1^2+y_1^2)(a^2y_1^2+b^2x_1^2-a^2b^2)$$
$$=(a^4y_1^2+b^4x_1^2)(\surd[a^2y_1^2+b^2x_1^2]+ab)^2\text{.........}(15),$$

the locus required. Equation (15) may be factored by the solution of Algebra problem 250, the relevant factor, with subscripts omitted, being

$$\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)^2=\left(\frac{a^2+b^2}{a^2-b^2}\right)^2\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right).$$

Also solved by G. B. M. Zerr.

281. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

In the proposition in solid geometry "If a line is perpendicular to each of two intersecting lines it is perpendicular to the plane of the lines," it is assumed that two intersecting lines have a common perpendicular. Prove it.

Solution by DR. GEORGE BRUCE HALSTED, Gambier, Ohio.

To prove that two intersecting straights have a common perpendicular.

To each of the given straights, a, b, at their intersection point A construct a perpendicular plane (Halsted's *Rational Geometry*, §337). They have a straight in common (H. R. G. I 6), which is perpendicular to a and perpendicular to b by H. R. G. §333, and that without *assuming* that any two intersecting straights have a common perpendicular.

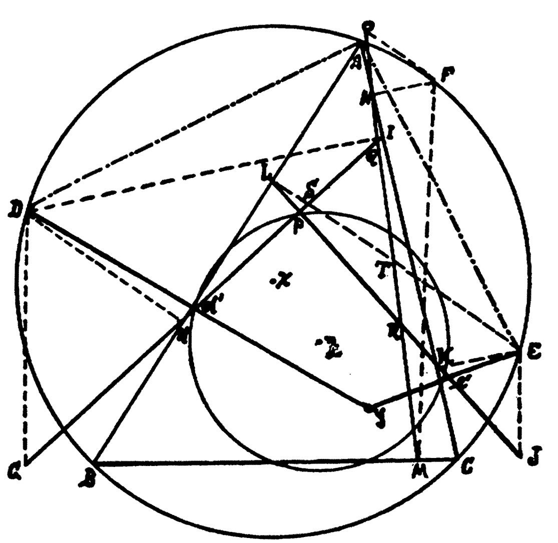
Also solved by A. H. Holmes, Rev. J. H. Meyer, and G. B. M. Zerr.

282. Proposed by REV. ALAN S. HAWKESWORTH, Allegheny, Pa.

The pedal lines of any two points on the circum-circle of a triangle concur in an angle equal to that subtended by the said points.

Solution by the PROPOSER.

Upon the circum-circle of triangle ABC take, first, any diameter DE. Then its pedal lines GHI and JKL will concur at P in a right angle.



Join DA, EA, and let GHI cut the perpendicular EL to AB in S. Then EKA and ELA being right angles, $EKLA$ are concyclic about EA; and angle $EAK=ELK$. Similarly, $DHIA$ are concyclic about DA; and $ADI=AHI$. Therefore the right angled triangles AID and SLH are similar; with angles DAI and LSP equal. And hence, even as the angles DAI and EAK are complementary, being within the semi-circle DAE; so also are their equals SLP and LSP. So that LPS is a right angle.